Int. J. Heat Mass Transfer. Vol. 7, pp. 527–529. Pergamon Press 1964. Printed in Great Britain

THE EQUATION OF A SQUARE IN HEAT TRANSFER

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(*Received* 20 *November* 1963)

Abstract—The purpose of this technical brief is to propose a formula which can replace advantageously the shape-factor formula because of its greater reliability. Steady state heat flow in the walls of an infinitely long square tunnel is evaluated with the proposed formula and results are compared with those obtained with the relaxation method [1].

NOMENCLATURE

N,	number of dimensions of a space;
x,	variable distance (ft);
C,	constant;
n,	exponent $(1 \leqslant n \leqslant \infty)$;
a, b, l.	constant distance (ft);
r,	distance from origin to x- or y-axis intercept (ft);
A,	area (ft^2);
$\dot{Q}$,	steady state heat flow (Btu/hr);
K,	thermal conductivity (Btu/hr ft degF);
T,	temperature (°F);
$f(n)$,	dimension coefficient equals to a function of $\Delta r/r_1$ [$f(n) \leqslant 8$];
$\Gamma(n)$,	gamma function.

GEOMETRICAL squares and cubes are members of the N-dimensional family [2–4] expressed by

$$\sum_{i=1}^{N} \left[\left(\frac{x_i}{a_i}\right)^2\right]^{n_i} = C \qquad (1)$$

in the above equation, a_i and C are constants with n_i a parameter. A particular case of equation (1) is the following family of pseudo-squares:

$$\sum_{i=1}^{2} x_i^{2n} = r^{2n}. \qquad (2)$$

In this equation r is the distance from the center to the x- or y-axis intercept, n is positive and greater than 1. It can be shown that the area of this family of curves is

$$A = \frac{4r^2\left\{\Gamma\left(\frac{1}{2n}+1\right)\right\}^2}{\Gamma\left(\frac{1}{n}+1\right)}. \qquad (3)$$

If we let $n = 1$ in equation (3) we get the area of a circle of radius r. When we differentiate the area of a circle with respect to r we get

$$\frac{dA}{dr} = 2\pi r \qquad (4)$$

which is the perimeter of a circle of radius r. Now, if we differentiate the area of a pseudo-square, we get

$$\frac{dA}{dr} = \frac{\partial A}{\partial r} + \frac{\partial A}{\partial n}\frac{dn}{dr} \qquad (5)$$

where

$$\frac{\partial A}{\partial r} = \frac{8\left\{\Gamma\left(\frac{1}{2n}+1\right)\right\}^2 r}{\Gamma\left(\frac{1}{n}+1\right)} \qquad (6)$$

and

$$\frac{dn}{dr} = 0, \text{ when } \begin{cases} n = 1 & \text{(concentric circles)} \\ n \to \infty & \text{(concentric squares)} \end{cases}. \qquad (7)$$

We can expect equation (6) to give the perimeters approximately when $n \geqslant 5$. The continuous curve in Fig. 1 shows the exact perimeter while the dotted line gives the approximated perimeter as a function of n.

Steady state two-dimensional heat flow can be expressed in its simpler form by

$$\dot{Q} = -KA\frac{dT}{dr}. \qquad (8)$$

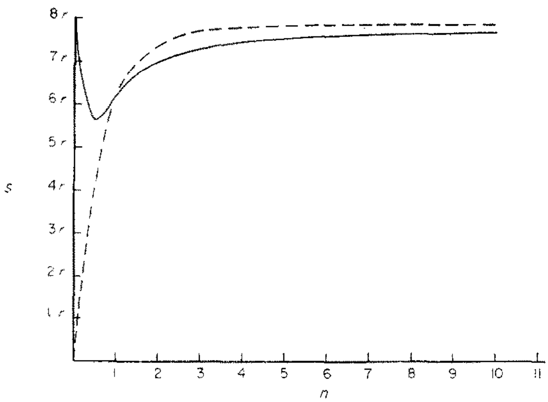

FIG. 1.

In this equation, $\dot{Q}$ is the heat flow per unit time, K the thermal conductivity, A the area perpendicular to the flow lines and dT/dr the temperature gradient. For two-dimensional heat flow in circular sections we get

$$\dot{Q} = \frac{2\pi K l \Delta T}{\ln\left(\frac{r_2}{r_1}\right)} \tag{9}$$

where l is the axial length. Equation (9) can be rearranged as follows:

$$\dot{Q} = \frac{f(n) K l \Delta T}{\ln\left(\frac{r_2}{r_1}\right)} \tag{10}$$

where in this case $f(n) = 2\pi$. Since circles and pseudo-squares belong to the same family of curves we can suspect that the area perpendicular to the flow-lines is

$$A \doteq l\frac{dA}{dr} = l\left[\frac{\partial A}{\partial r} + \frac{\partial A}{\partial n}\cdot\frac{dn}{dr}\right] \tag{11}$$

which reduces to

$$A \doteq l\left[\frac{8\left\{\Gamma\left(\frac{1}{2n}+1\right)\right\}^2 r}{\Gamma\left(\frac{1}{n}+1\right)}\right] + l\frac{\partial A}{\partial n}\frac{dn}{dr}. \tag{12}$$

If we substitute equation (12) in equation (8) we might obtain a general formula for the heat loss through a family of pseudo-squares including circles as a particular case.

Considering the isothermal patterns in the walls of a square tunnel we can suppose that they are pseudo-squares and that the heat flows through a series of such concentric pseudo-squares separated by a nearly constant distance. Hence equation (10) which is exact for circular sections could be used to estimate the heat loss through these isotherms. Figure 2 shows that $f(n)$ varies slightly with the ratio obtained from wall thickness over inside dimension. Table 1 illustrates a few numerical problems with $K = 1$ unit.

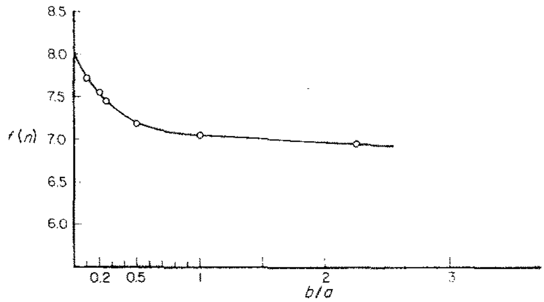

FIG. 2.

Table 1

Inside dimensions a (ft)	Wall thickness b (ft)	Temperature difference (degF)	Heat flow (relaxation) $\dot{Q}\left(\frac{\text{Btu}}{\text{h ft}}\right)$	$f(n)$	b/a
40	4	600	25 472	7·72	0·1
20	4	600	13 472	7·55	0·2
16	4	600	11 302	7·45	0·25
12	6	600	6 222	7·19	0·5
4	4	600	3 888	7·06	1·0
5	2	600	2 332	6·95	2·25

Example

Let us solve the problem of p. 367, in Jacob, *Heat Transfer*, Vol. I, John Wiley (1958), which can be summarized as follows:

Find the heat loss per foot through the walls of a square furnace 44 in outside dimension, 24 in inside dimension, with $K = 1·5$ unit and $\Delta T = 400$ degF.

Here $\frac{b}{a} = \frac{10}{24} = 0{\cdot}416$, from Fig. 2, we find $f(n) \doteq 7{\cdot}22$.

$$\therefore \frac{\dot{Q}}{L} \doteq \frac{7{\cdot}22 \times 1{\cdot}5 \times 400}{\ln\left(\frac{22}{12}\right)} = 7200 \text{ Btu/hr ft}$$

which is very closed to the answer given 7203 Btu/hr ft.

It is clear that this proposed method cannot be preferred to the relaxation method when computers and programmers are available. However, engineers shall find in it a handy and costless way to estimate rapidly heat losses for insulation problems involving square sections.

ACKNOWLEDGEMENT

The author wishes to express his thanks to Prof. J. Bazinet of the University of Sherbrooke for the results obtained with the relaxation method and to Prof. F. Goodspeed of Université Laval for helpful suggestions concerning this technical brief.

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Zusammenfassung—Es wird hier eine Formel vorgeschlagen, die wegen ihrer grösseren Zuverlässigkeit die Formfaktorgleichung ersetzen könnte. Der stationäre Wärmefluss in den Wänden eines unendlich langen Rechtecktunnels wird mit der vorgeschlagenen Formel berechnet und die Ergebnisse mit jenen verglichen, die man nach der Relaxationsmethode erhält [1].

Аннотация—Цель данного краткого технического обзора предложить формулу, которая благодаря большой надежности могла бы удачно заменить формулу для коэффициента формы. Установившийся тепловой поток в стенках бесконечно длинного канала квадратного сечения рассчитывается по предложенной формуле, а результаты сравниваются с результатами, полученными по релаксационному методу.